

Ejercicios de cálculo

① Derivadas

$$a) f(x) = \sin x^2 \quad \begin{array}{c} \text{regla cadena} \\ \downarrow \end{array} \quad f'(x) = \cos(x^2) \cdot 2x \quad \begin{array}{c} \text{reordenar} \\ \downarrow \end{array} = 2x \cos x^2 \quad \underline{\underline{B}}$$

$$b) f(x) = \sin^2 x; \quad f'(x) = 2 \sin(x) \cdot \cos(x) \quad \underline{\underline{B}}$$

$$c) f(x) = (3x^2 - 2)^5; \quad f'(x) = 5(3x^2 - 2)^4 \cdot (6x) = 30x(3x^2 - 2)^4 \quad \underline{\underline{B}}$$

$$d) f(x) = \sqrt[3]{x^2 - 3}; \quad f'(x) = \frac{1}{3} (x^2 - 3)^{-\frac{2}{3}} \cdot 2x = \frac{2x}{3 \sqrt[3]{(x^2 - 3)^2}} \quad \underline{\underline{B}}$$

$$e) f(x) = \frac{1}{7x+1}; \quad f'(x) = \frac{0 \cdot (7x+1) - 1 \cdot 7}{(7x+1)^2} = \frac{-7}{(7x+1)^2} \quad \underline{\underline{B}}$$

$$f) f(x) = x \cos 2x; \quad f'(x) = 1 \cdot \cos 2x + x \cdot (-\sin 2x) \cdot 2 = \\ = \cos 2x - 2x \sin 2x \quad \underline{\underline{B}}$$

② Integrales

$$a) \int x^5 dx = \frac{1}{6} x^6 = \frac{x^6}{6} + C$$

$$b) \int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx = \int \frac{1}{x^2} dx + \int \frac{4}{x\sqrt{x}} dx + \int 2 dx =$$

$$= \int x^{-2} dx + \int 4$$

Ejercicios de Trigonometría

① Pase a grados

$$\text{Factor: } \pi \text{ rad} = 180^\circ$$

$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.296$$

a) $1 \text{ rad} = 57,296^\circ$ B

b) $3 \text{ rad} = 171,88^\circ$ B

c) $5 \text{ rad} = 286,47^\circ$ B

② Pase a rad

$$\text{Factor: } 180^\circ = \pi \text{ rad}; 1^\circ = \frac{\pi \text{ rad}}{180}$$

a) $50^\circ = \frac{50 \pi}{180} \approx 0.872 \text{ rad}$ B

b) $70^\circ = \frac{70 \pi}{180} = 1,22 \text{ rad}$ B

c) $110^\circ = \frac{110 \pi}{180} = 1.919 \text{ rad}$ B

③

$$\cos \theta = 0,46 \quad \text{y } \theta \text{ en } 1^\circ \text{ cuadrante}$$

Con calculadora

~~$$\arccos 0,46$$~~

$$\arccos 0,46 = 62,61^\circ \text{ en } 1^\circ \text{ cuadrante}$$

Nota: No puede ser $242,61$ por estar en el 4.º cuadrante

④ $\cos \theta = -0,46$, θ en 2.º cuadrante

$$\arccos -0,46 \approx 163,74^\circ$$

Nota: no vale $253,74$ por ser del 3.º cuadrante

⑤ $\text{sen } \theta = 0,83$ y θ en 1° cuadrante

$$\text{arc sen } 0,83 \approx 56,09^{\circ} \quad \underline{\underline{B}}$$

No vale $146,09$ por ser del 2° cuadrante

⑥ $\text{sen } \theta = -0,52$ y θ en 4° cuadrante

$$\text{arcsen } -0,52 = -31,33 \xrightarrow{\text{sum } 360^{\circ}} = 328,66 \quad \underline{\underline{B}}$$

Complejos

① Pasar a forma polar

a) $z_1 = 3 + 2j$

$$r = \text{Módulo} = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$$

$$\theta = \text{Argumento (fase)} = \arctan\left(\frac{2}{3}\right) \approx 33,69^\circ$$

$$z_1 = \sqrt{13} \angle 33,69^\circ = 3,606 \angle 33,69^\circ \quad \underline{B}$$

b) $z_2 = 2 - 5j$

$$\text{Módulo} = \sqrt{2^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29}$$

$$\text{Argumento (fase)} = \arctan\left(\frac{-5}{2}\right) \approx -68,19^\circ$$

$$z_2 = \sqrt{29} \angle -68,19^\circ = 5,385 \angle -68,19^\circ \quad \underline{B}$$

c) $z_3 = 1 - 2j$

$$r = \sqrt{1^2 + (-2)^2} = \sqrt{1 + 4} = \sqrt{5} \approx 2,236$$

$$\theta = \arctan\left(\frac{-2}{1}\right) = \arctan -2 \approx -63,44^\circ$$

$$z_3 = 2,236 \angle -63,44^\circ \quad \underline{B}$$

② Pasa a forma cartesiana

$$\begin{aligned} a) z_1 &= 5 \angle 30^\circ = 5 \cos 30^\circ + 5 \sin(30^\circ)j = \\ &= \frac{5\sqrt{3}}{2} + \frac{1}{2}5j \simeq 4.330 + \cancel{2.5} 2.5j \quad \underline{\underline{B}} \end{aligned}$$

$$\begin{aligned} b) z_2 &= 27 \angle 60^\circ = 27 \cos 60^\circ + 27 \sin(60^\circ)j = \\ &= \frac{27}{2} + 27 \cdot \frac{\sqrt{3}}{2}j = 13.5 + 11.691j \quad \underline{\underline{B}} \end{aligned}$$

$$\begin{aligned} c) z_3 &= 10 \angle -20^\circ = 10 \cos(-20) + 10 \sin(-20)j = \\ &= 9.397 + (-3.420)j = 9.397 - 3.420j \quad \underline{\underline{B}} \end{aligned}$$

~~9.397~~
~~10.24~~
~~2.3~~
~~z₁~~

③

$$z_1 = 2 + 2j = 2\sqrt{2} \angle 45^\circ \simeq 2.828 \angle 45^\circ$$

$$z_2 = 1 - 3j = \sqrt{10} \angle -71.56^\circ = 3.16 \angle -71.56^\circ$$

$$z_3 = 4 - 5j = \sqrt{41} \angle -51.34^\circ$$

$$z_4 = -2 + 3j = \sqrt{13} \angle 123.69^\circ$$

$$a) z_1 + z_2 = 2 + 1 + (2 - 3)j = 3 - j = 3 - j$$

$$b) z_1 - z_3 = 2 - 4 + [2 - (-5)]j = -2 + 7j$$

$$c) z_4 - z_1 = -2 - 2 + (3 - 2)j = -4 + j$$

$$d) z_1 \cdot z_2 = (2\sqrt{2} \cdot 3) \angle (45 - 71.56)^\circ = 6\sqrt{2} \angle -26.56^\circ$$

$$\simeq 6\sqrt{2} \cos(-26.56) + 6\sqrt{2} \sin(-26.56)j = 7.58 - 3.79j$$

Mal porque calculé mal la forma polar de z_2

$$z_1 \cdot z_2 = (2\sqrt{2} \sqrt{10}) \angle 45 - 71.56 = 8 - 4j$$

$$e) z_1 z_3 = 18 - 2j$$

$$f) z_4 \cdot z_2 = 7 + 9j$$

$$g) \frac{z_2}{z_1} = -\frac{1}{2} - j$$

$$h) \frac{z_4}{z_3} = 0.561 + 0.048j$$

$$i) \frac{z_3}{z_2} = 1.9 + 0.7j$$